ISSN 2661-2666 (Online) International Scientific Journal Monte (ISJM) DOI: 10.33807/monte.20222618
Volume 6, (No).2 (2022): April

**Sead Rešić**
Faculty of Science, Department of mathematics, University of Tuzla
Univerzitetska 4, Bosnia and Herzegovina
E-mail: sresic@hotmail.com
**Husnija Bibuljica**
Faculty of Economics, International University Travnik" Travnik,Bosnia and Hercegovina
E-mail: husnija.bibuljica @iu-travnik.com

**Arnes Z. Hadžiomerović**
Second Gymnasium Mostar
USRC "Midhad Hujdur Hujka" bb, Mostar, Bosnia and Herzegovina
E-mail: aneshagi@gmail.com

# Three different approaches to solving the problem of multicriteria fractional linear programming

## Abstract

In this paper, we propose a new approach to target programming based the linearization fractured linear target functions for problem solving fuzzy multiple criteria fractured linear programming. By the proposed method, decision makers need to give the information to the relative importance of target function. The applicability of the proposed method has been tested on the example of the financial structure optimization of a company. The results show the advantages of the proposed methodology in comparison with existing methods: (a) The methodology is simple for analysts as well as for decision-makers, and (b) the decision makers can determine the weight of the target function, and thereby the resulting solutions will reflect the preferences of decision-makers.

*Keywords*: Multicriteria, linear fractional, targeted programming.

## 1. Introduction

In some economic problems, goals may be more appropriately expressed as a ratio of two economic magnitudes. Economic problems presented in this way can better reflect the quality of business results. Also, the goals expressed in this way allow us to make adequate comparisons between the two business entities. Therefore, if the goals are expressed as a ratio of two economic magnitudes and if the parameters and variables of the model are linear, then the optimization of the economic problem requires multicriteria fractional linear programming. (MOFLP).

The problem of fractional linear programming with one goal was extensively researched in the second part of the twentieth century and successful methods have been developed to solve such problems ([3], [4], [5], [11]).

However, in MOFLP problems determining a successful (Pareto optimal) solution is technically demanding when the target functions are fractionally linear. Solving a multicriteria fractional linear programming model is limited to a small number of multicriteria programming methods that are not effective enough either from the point of view of analysts or decision makers [1], [2], [6], [10], [12], [13], [21]).

A particular problem grows in the application of target programming methods when deviation variables $d^-$ and $d^+$ fractional linear functions are added to form constraints to the target programming model, because then nonlinear constraints are obtained that lead to model solving problems. Several methods have been developed that use target programming to solve a multicriteria fractional linear programming model ([8], [9], [15], [16], [17], [20], [22]), but there is little research in which these methods have been applied and tested in solving real economic problems ([13], [20]).

Some papers propose linearization of fractional linear objective functions and solve the thus obtained multicriteria linear programming model using standard multicriteria linear programming methods such as target programming, STEM method, fuzzy programming, etc. ([17], [19]). These methods are not efficient enough, neither from the point of view of analysts nor decision makers. In this paper, a new target programming based on the linearization of fractional linear target functions for solving unclear multicriteria fractional programming models is presented. In the proposed method, the decision maker provides information on the relative importance of the target functions. The proposed method was tested on the example of optimizing the financial structure of a company.

Therefore, the main objectives of this paper are: (1) to propose a model based on linearization of objective functions to improve target programming methods designed to solve multicriteria fractional linear programming, (2) to test the applicability of the proposed model to solve financial structure optimization problems.

## 2. Methods for solving problems of fractional-linear programming

We can apply a number of methods to solve the model of fractional linear programming. When choosing the method, we took into account that the simplex method can be used when solving the model. We have narrowed the choice of methods to the following three groups of methods:

1. Method of meeting the objectives,
2. Targeted programming,
3. 'Fuzzy' programming.

### 2.1. Goal satisfaction method

The goal satisfaction method belongs to the group of general methods of multicriteria programming. It is suitable for solving all classes of multicriteria programs, so it can also be used for solving multicriteria models of fractionally linear programming.

This method was proposed by Benson [1975] for the interactive solution of linear and nonlinear multicriteria programming models. In this method, the decision maker indicates a set of criterion levels $L_j,\ j=1,\cdots,k$ (which must be permissible) and determines one function of the criterion, the level of which is the least satisfied (LS). The LS criterion function is then maximized subject to the original constraints and additional constraints formed by other criterion functions. An analyst, working iteratively and interactively, can reduce the value of acceptable criterion levels one or more times until the most favorable solution for the decision maker is reached.

The algorithm of this method consists of the following steps:

**Step 0:**

The decision maker determines a set of minimum acceptable criterion levels $\underline{L}^1$, which will serve as a starting point for a later revision of the value of the criterion functions. The set $\underline{L}^1$ and constraints of the model should form the allowable area in step 2, otherwise the decision maker must perform a value audit $\underline{L}^1$. We set $q$ = 1.

**Step 1:** Selection of the criterion function that is least satisfied

The decision maker determines one criterion function that is least met (LS). This is the function in which the values of the criterion functions for a particular optimal (marginal) solution differ the most.

**Step 2:** Optimization of LS function criteria

The LS function is maximized under the condition of the original constraints and additional constraints obtained from the remaining criterion functions:

$$\begin{aligned} &\max f = f_{LS}(\underline{x}) \\ &\text{u.o.} \\ &g_i(\underline{x}) \le 0, \qquad i=1,\cdots,m \\ &f_j(\underline{x}) \ge L_j^q, \qquad j=1,\cdots,k, \quad j \ne LS\,. \end{aligned} \tag{2.10}$$

**Step 3:** Decision-making phase

The decision maker indicates whether the achievement $f_{LS}(\underline{x})$ in step 2 is satisfactory. (a) If not, the decision maker changes some levels of criterion functions and determines $\underline{L}^{q+1}\ (\le \underline{L}^q)$. Let $q = q + 1$. We return to step 2. (b) If the achievement $f_{LS}(\underline{x})$ is satisfactory, the decision maker is asked whether the achieved value $f_{LS}(\underline{x})$ can be somewhat weakened in order to improve the levels of other criteria. If it cannot be weakened, then $\underline{L}^q$ and the optimized value are the best achieved criterion function of the model. Otherwise you need to go to step 4.

**Step 4:** Determining a new level of criterion function

The decision maker specifies the amount of mitigation $\Delta f_{LS}(\underline{x})$, which is permissible. Take $q = q + 1$. Go back to step 1.

If the model defines that the function $f_{LS}(\underline{x})$ is nonlinear, and / or some constraints are nonlinear, this model can be solved by applying the appropriate nonlinear programming method.

## 2.2. Targeted Programming

Target programming as an approach was first introduced by Charnes and Cooper (1961), then developed by Ijiry (1965), Lee (1972), Ignizio (1982), etc. The basic idea of target programming is to minimize the distance between the vector function $Z$ (how is defined in model (3.1)) and aspiration levels of the vector $\overline{Z}$. The aspiration level of $\overline{Z}$ is determined by the decision maker or is equal to Z*, where Z* = $(z_1^*, z_2^*, ..., z_K^*)$.

In the target programming, the distance between $Z_k$ i $\overline{Z}_k$, $d(Z_k, \overline{Z}_k)$,) is expressed by the deviation variables $n_k$ and $p_k$ ($k = 1, 2,\ldots, K$) where $n_k$ are negative deviation variables,

$$n_k = \max(0, \overline{z}_k - z_k) = \frac{1}{2}\left[\overline{z}_k - z_k + \left|\overline{z}_k - z_k\right|\right], \tag{2.11}$$

and $p_k$ are positive deviation variables,

$$p_k = \max(0, z_k - \overline{z}_k) = \frac{1}{2}\left[z_k - \overline{z}_k + \left|z_k - \overline{z}_k\right|\right]. \tag{2.12}$$

Minimizing the distance between $z_k$ and $\overline{z}_k$ means minimizing either $n_k$ or $p_k$ or $n_k + p_k$. For the problem of maximum it should be $z_k \geq \overline{z}_k$, so it is necessary to minimize $n_k$, while for the problem of minimum $z_k \leq \overline{z}_k$, it is necessary to minimize $p_k$. When $z_k = \overline{z}_k$ it is necessary to minimize $n_k + p_k$.

According to the above, model (2.13) was converted into the minimization problem of deviation variables with the help of target programming and can have one of the following forms:

(i) Min-Max form:

(M2)

$$\min \max g_k(n_k, p_k) \tag{2.14}$$

$$\text{p.o.} \quad \underline{A}\ \underline{X} \begin{pmatrix} \leq \\ = \\ \geq \end{pmatrix} \underline{b}, \tag{2.15}$$

$$\underline{C}_k \underline{X} + n_k - p_k = \overline{z}_k,\ k = 1, 2, \ldots, K, \tag{2.16}$$

$$\underline{X} \geq 0,\ n_k \geq 0,\ p_k \geq 0,\ n_k \cdot p_k = 0,\ k = 1, 2, \ldots, K, \tag{2.17}$$

where $g_k(n_k, p_k) = n_k$ in case of maximization $z_k$, $g_k(n_k, p_k) = p_k$ in case of minimization $z_k$, and $g_k(n_k, p_k) = n_k + p_k$ when $z_k = \overline{z}_k$, $\underline{C}_k$ is the $k$-th row of the matrix $\underline{C}$.

Model 2 is converted to the LP problem as follows:

(M3)

$$\min \lambda \tag{2.18}$$

$$\text{u. o. restrictions (2.15) do (2.17),} \tag{2.19}$$

$$\lambda \geq g_k(n_k, p_k),\ k = 1, 2, \ldots, K. \tag{2.20}$$

(M3) can be solved by the simplex method.

(ii) Minimization of the sum of deviation forms:

(M4)

$$\min \sum_{k=1}^{K} g_k(n_k, p_k) \tag{2.21}$$

$$\text{u. o. restrictions (2.15) – (2.17)} \tag{2.22}$$

where all parameters and variables are defined in (M1) and (M2). (M4) is a linear programming (LP) problem that can be solved by the simplex method.

(iii) Minimization of the weight sum of deviation forms:

(M5) $$\min \sum_{k=1}^{K} w_k g_k(n_k, p_k) \quad (2.23)$$

u. o. restrictions (2.15) – (2.17), (2.24)

where $w_k$ ($k$ = 1, 2, …, $K$) are determined by the decision maker. (M5) is an LP problem that can be solved by the simplex method.

(iv) Lexicographic form: If the decision maker can rank the target functions by priority, then the lexicographic form of target programming can be used. In this form $K$ of the target functions is arranged in order of priority. The goal with the highest priority is considered first, then the goal with priority to the highest, etc. The lexicographic form of target programming looks like this:

(M6) $$\min a = \left\{ \sum_{k \in P_i} w_k g_k(n_k, p_k) : \ i = 1, 2, ..., I \right\} \quad (2.25)$$

u. o. restrictions (2.15) – (2.17), (2.26)

where $I$ is the priority level number, and $k \in P_i$ means that the $k$-th goal is at the $i$-th priority level.

(M6) is a linear target programming model that can be solved using a multiphase simplex method or a sequential simplex method.

In models (MF1), (MF2), (MF3) and (MF4) broken linear functions $z_k(x) = \dfrac{\underline{C}_k x + c_0^k}{\underline{D}_k x + d_0^k}$ are transformed into linear functions.

To solve the multicriteria problem of fractionally linear programming with vaguely defined goals of the decision maker using the method of target programming, we propose a modification of the linearization method published in the paper (Pal et al., 2003). Since the decision maker is not able to accurately express the value of the goals to be achieved, it is proposed here to determine the goals of the decision maker using a payment table, which is formed by optimizing each individual goal function on a given set of constraints. In goal functions to be maximized, the upper limit is the maximum value of goal functions obtained by maximizing them on a given constraint set, while the lower limit is the smallest value of a given goal function on a given constraint set. In the goal functions to be minimized, the upper limit is the highest value of the goal function from the payment table, while the lower limit is the minimum value of the goal function obtained by minimizing on a given set of constraints.

The proposed modification of the method requires the decision maker to provide information on the difficulty of the goal functions. If the decision maker cannot express the relative importance of the goal functions, the weights can be calculated using one of the methods for determining the weights of the goal functions. In order to solve the model of multicriteria fractional linear programming with imprecisely determined goals of the decision maker using the method of target programming, for the goal functions to be maximized it is proposed to linearize the fractional linear goal function as follows:

$$\frac{\sum_{j=1}^{n} c_{kj} x_j + c_0^k}{\sum_{j=1}^{n} d_{kj} x_j + d_0^k} \le z_k^* \, / \cdot \left( \sum_{j=1}^{n} d_{kj} x_j + d_0^k \right), \ k = 1, 2, \ldots, k_1$$

$$\sum_{j=1}^{n} c_{kj} x_j + c_0^k \le z_k^* \left( \sum_{j=1}^{n} d_{kj} x_j + d_0^k \right)$$

ISSN 2661-2666 (Online) International Scientific Journal Monte (ISJM) DOI: 10.33807/monte.20222618
Volume 6, (No).2 (2022): April

$$\sum_{j=1}^{n} c_{kj} x_j - z_k^* \sum_{j=1}^{n} d_{kj} x_j \le z_k^* d_0^k - c_0^k$$

$$\sum_{j=1}^{n} (c_{kj} - z_k^* d_{kj}) x_j \le z_k^* d_0^k - c_0^k$$

$$\sum_{j=1}^{n} C_{kj} x_j + d_k^- = Z_k^*, \;\; k = 1, 2, ..., k_1, \qquad (2.27)$$

Where $z_k^*$ is the maximum value of the k-th function of the target on a given set of constraints, $C_{kj} = c_{kj} - z_k^* d_{kj}$, $Z_k^* = z_k^* d_0^k - c_0^k$, a $d_k^-$ is a deviation variable ($k$ = 1, 2, ..., $k_1$).

A fractionally linear function that needs to be minimized can be linearized in an analogous way:

$$\frac{\sum_{j=1}^{n} c_{kj} x_j + c_0^k}{\sum_{j=1}^{n} d_{kj} x_j + d_0^k} \ge z_k^* / \cdot \left( \sum_{j=1}^{n} d_{kj} x_j + d_0^k \right)$$

$$\sum_{j=1}^{n} c_{kj} x_j + c_0^k \ge z_k^* (\sum_{j=1}^{n} d_{kj} x_j + d_0^k)$$

$$\sum_{j=1}^{n} c_{kj} x_j - z_k^* \sum_{j=1}^{n} d_{kj} x_j \ge z_k^* d_0^k - c_0^k$$

$$\sum_{j=1}^{n} (c_{kj} - z_k^* d_{kj}) x_j \ge z_k^* d_0^k - c_0^k$$

$$\sum_{j=1}^{n} C_{kj} x_j - d_k^+ = Z_k^*, \;\; k = k_1, k_1 + 1, ..., K, \qquad (2.28)$$

Where $Z_k^*$ is the minimum value of the k-th target function on a given constraint set, and $d_k^+$ is a deviation variable ($k = k_1 + 1, k_1 + 2, ..., K$).

Based on the above, the targeted programming model looks like:

$$\min g_k ( d_k^-, d_k^+ )$$

$$\text{u.o.} \quad C_{kj} x_j + d_k^- = Z_k^*, \;\; k = 1, 2, ..., k_1,$$

$$C_{kj} x_j - d_k^+ = Z_k^*, \;\; k = k_1, k_1 + 1, ..., K,$$

$$Ax \begin{pmatrix} \le \\ = \\ \ge \end{pmatrix} b, \qquad (2.29)$$

$$x \ge 0,$$

$$D_k^-, D_k^+ \ge 0.$$

Numerous target programming approaches can be used to solve the model (2.39) to solve the model of broken linear target programming:

(1) Min-max access:

$$\text{Min max } g_k(d_k^-, d_k^+)$$

u.o.

$$\sum_{j=1}^{n} C_{kj} x_j + d_k^- = Z_k^*, \ \ k = 1, 2, ..., k_1,$$

$$\sum_{j=1}^{n} C_{kj} x_j - d_k^+ = Z_k^*, \ \ k = k_1, k_1 + 1, ..., K,$$

$$Ax \begin{pmatrix} \le \\ = \\ \ge \end{pmatrix} b, \qquad (2.30)$$

$$x \ge 0, \ d_k^-, d_k^+ \ge 0.$$

Therefore, $d_k^-$ appears within the constraints formed by the goal functions to be maximized, while $d_k^+$ appears within the constraints formed by the goal functions to be minimized.

To solve the model (2.30) we can form the following specific model of linear programming, which is simply solved by the simplex method:

$$\text{Min } \lambda$$

s. t.

$$\lambda - d_k^- \ge 0, k = 1, 2, ..., k_1,$$

$$\lambda - d_k^+ \ge 0, k = k_1 + 1, k_1 + 2, ..., K,$$

$$\sum_{j=1}^{n} C_{kj} x_j + d_k^- = Z_k^*, \ \ k = 1, 2, ..., k_1,$$

$$\sum_{j=1}^{n} C_{kj} x_j - d_k^+ = Z_k^*, \ \ k = k_1, k_1 + 1, ..., K,$$

$$Ax \begin{pmatrix} \le \\ = \\ \ge \end{pmatrix} b, \qquad (2.31)$$

$$x \ge 0, \ \lambda, d_k^-, d_k^+ \ge 0.$$

(2) Minimization of the sum of deviation forms:

$$\text{Min } \sum_{k=1}^{K} g_k(d_k^-, d_k^+)$$

u.o.

$$\sum_{j=1}^{n} C_{kj} x_j + d_k^- = Z_k^*, \ \ k = 1, 2, ..., k_1,$$

$$\sum_{j=1}^{n} C_{kj} x_j - d_k^+ = Z_k^*, \ \ k = k_1, k_1 + 1, ..., K,$$

$$Ax\begin{pmatrix}\le\\=\\\ge\end{pmatrix}b, \qquad (2.32)$$

$$x\ge 0,\ d_k^-, d_k^+ \ge 0.$$

Model (2.32) is a linear programming model that is also easily solved by the simplex method.

(3) Min-max weight form:

$$\text{Min } g_k(w_k d_k^-, w_k d_k^+)$$

$$\text{u.o.} \quad \sum_{j=1}^{n} C_{kj}x_j + d_k^- = Z_k^*,\ k=1,2,\dots,k_1,$$

$$\sum_{j=1}^{n} C_{kj}x_j - d_k^+ = Z_k^*,\ k=k_1,k_1+1,\dots,K,$$

$$Ax\begin{pmatrix}\le\\=\\\ge\end{pmatrix}b, \qquad (2.33)$$

$$x\ge 0,\ d_k^-, d_k^+ \ge 0.$$

where $w_k$, $\sum_{k=1}^{K} w_k = 1$, ($k$ = 1, 2, …, $K$) are the weightiness of the goal functions determined by the decision maker.

To solve model (2.33) we can form the following model of linear programming, which is solved by the simplex method:

$$\text{Min } \lambda$$

$$\text{s. t. } \lambda - w_k d_k^- \ge 0,\ k=1,2,\dots,k_1,$$

$$\lambda - w_k d_k^+ \ge 0,\ k=k_1+1,k_1+2,\dots,K,$$

$$\sum_{j=1}^{n} C_{kj}x_j + d_k^- = Z_k^*,\ k=1,2,\dots,k_1,$$

$$\sum_{j=1}^{n} C_{kj}x_j - d_k^+ = Z_k^*,\ k=k_1,k_1+1,\dots,K,$$

$$Ax\begin{pmatrix}\le\\=\\\ge\end{pmatrix}b, \qquad (2.34)$$

$$x\ge 0,\ \lambda, d_k^-, d_k^+ \ge 0.$$

(iv) Lexicographic form: In this form $K$ goal functions are arranged according to the priorities of the decision maker, where the objective with the highest priority is considered first, then the second, etc. The general form of the lexicographic approach of target programming is

$$\min a = \min_{k\in P_i} g_k(d_k^-, d_k^+) : i=1,2,\dots,K;\ k=1,2,\dots,K$$

$$\text{s.t.} \quad \sum_{j=1}^{n} C_{kj} x_j + d_k^- = Z_k^*, \;\; k = 1, 2, ..., k_1,$$

$$\sum_{j=1}^{n} C_{kj} x_j - d_k^+ = Z_k^*, \;\; k = k_1, k_1 + 1, ..., K,$$

$$Ax \begin{pmatrix} \leq \\ = \\ \geq \end{pmatrix} b, \qquad (2.35)$$

$$x \geq 0, \;\; d_k^-, d_k^+ \geq 0,$$

where $I$ is the priority level number, and $k \in P_i$ means that the $k$-th goal is at the $i$-th priority level.

Model (2.35) is a model of linear programming that is solved by the simplex method.

### 2.3. 'Fuzzy' multicriteria fractional linear programming

If there is an inaccurately determined aspiration level for each goal function in the multicriteria fractional linear programming model, then these 'fuzzy' goal functions are expressed as 'fuzzy' goals.

Let $g_k$ be the aspiration level of the k-th function $z_k(x)$. Then the ‘fuzzy’ goals are expressed as

(1) $z_k(x) >\approx g_k$ (to maximize $z_k(x)$);

(2) $z_k(x) <\approx g_k$ (to minimize $z_k(x)$),

where $>\approx$ i $<\approx$ indicate (‘fuzzy’) defined aspiration levels.

'Fuzzy' fractional linear target programming can be presented as:

Find $x$

So they are satisfied $z_k(x) >\approx g_k$, $k = 1, 2, ..., k_1$

$$z_k(x) <\approx g_k, \; k = k_1 + 1, k_1 + 2, ..., K \qquad (2.36)$$

with restrictions $Ax \begin{pmatrix} \leq \\ = \\ \geq \end{pmatrix} b,$

$$x \geq 0.$$

Now 'fuzzy' goals are characterized by their ‘membeship’ functions. 'Membership' functions $\mu_k$ for *k-th* 'fuzzy' goal $z_k(x) >\approx g_k$ can be defined as:

$$\mu_k(x) = \begin{cases} 1 & \text{ako je} \quad z_k(x) \geq g_k \\ \dfrac{z_k(x) - l_k}{g_k - l_k} & \text{ako je} \quad l_k \leq z_k(x) \leq g_k \\ 0 & \text{ako je} \quad z_k(x) \leq l_k, \end{cases} \qquad (2.37)$$

Where $l_k$ is the lower limit for $k$-th 'fuzzy' goal.

This ‘membership’ function $\mu_k$ can be expressed for $k$-th 'fuzzy' goal $z_k(x) <\approx g_k$ as:

$$\mu_k(x)=\begin{cases}1 & \text{if} \quad z_k(x)\le g_k\\ \dfrac{u_k-z_k(x)}{u_k-z_k} & \text{if} \quad g_k\le z_k(x)\le u_k\\ 0 & \text{if} \quad z_k(x)\ge u_k,\end{cases} \tag{2.38}$$

where $u_k$ is the upper limit of tolerance.

Since in the 'fuzzy' programming approaches the highest degree of 'membership' function is equal to 1, for the defined 'membership' functions in (2.37) and (2.38), flexible 'membership' goals with aspiration level 1 can be represented as:

$$\frac{z_k(x)-l_k}{g_k-l_l}+d_k^- - d_k^+ =1, \tag{2.39}$$

$$\frac{u_k-z_k(x)}{u_k-g_k}+d_k^- - d_k^+ =1, \tag{2.40}$$

where $d_k^-$ $(\ge 0)$ i $d_k^+$ $(\ge 0)$ sa $d_k^- \cdot d_k^+ =0$ represent negative and positive deviations from given levels, respectively.

In this approach, only negative deviation variables should be minimized to achieve given levels of 'fuzzy' goals.

Since membership goals (2.39) and (2.40) are nonlinear in nature, this can cause computational difficulties in the resolution process. To avoid computational difficulties, linearization of 'membership' functions has been proposed.

*k*-th 'membership' goal in (2.39) can be expressed as

$$L_k z_k(x)-L_k l_k + d_k^- - d_k^+ =1, \text{ where } L_k=\frac{1}{g_k-l_k}.$$

By introducing expressions $z_k(x)$ from (2.36), the above goal can be represented as

$$L_k(c_k x+c_0^k)+d_k^-(d_k x+d_0^k)=L_k^{'}(d_k x+d_0^k), \text{ gdje je } L_k^{'}=1+L_k l_k,$$

or

$$C_k x+d_k^-(d_k x+d_0^k)-d_k^+(d_k x+d_0^k)=G_k, \tag{2.41}$$

where $C_k=L_k c_k - L_k^{'} d_k$, $G_k=L_k^{'} d_0^k - L_k c_0^k$.

In a similar way, goal expressions for 'membership' goals are obtained in (3.50).

Using the variable substitution method, the goal expressions from (3.51) can be linearized:

$$C_k x+D_k^- - D_k^+ =G_k,$$

where $D_k^- = d_k^-(d_k x+d_0^k)$ and $D_k^+ = d_k^+(d_k x+d_0^k)$, $D_k^-, D_k^+\ge 0$, $D_k^-\cdot D_k^+ =0$ since it is $d_k^-, d_k^+\ge 0$ and $d_k x+d_k^0>0$.

Since minimization $d_k^-$ means minimization $D_k^-/(d_k x+d_0^k)$, i $d_k^-=0$ when the 'membership' goal is fully achieved, and $d_k^-\le 1$ when the 'membership' goal is not fully achieved, inclusion $d_k^-\le 1$ in the solution leads to the introduction of the following constraint in the model:

$$\frac{D_k^-}{d_k x+d_0^k}\le 1, \text{ to jest } -d_k x+D_k^-\le d_0^k.$$

Thus, any such restriction corresponding to $d_k^+$ does not appear in the model formulation.

If 'min-sum target programming' is introduced into the model formulation, then the target programming model formulation becomes:

Find $x$ so that is

$$\text{Min } z = \sum_{k=1}^{K} w_k^- D_k^-$$

So it is satisfied $C_k x + D_k^- - D_k^+ = G_k$

$$\text{with restrictions} \quad Ax \begin{pmatrix} \leq \\ = \\ \geq \end{pmatrix} b \tag{2.42}$$

and $-d_k x + D_k^- \leq d_0^k$,

$x \geq 0,$

$D_k^-, D_k^+ \geq 0,$

Where $z$ represents the function of complete realization, which consists of negative deviation variables, and numerical weights $w_k$ represent the relative importance of achieving aspirational levels of 'fuzzy' goals on a set of constraints. The weighting scheme proposed by Mohamed (1996) can be used to determine the values $w_k^-$ ($k$ = 1, 2, ..., $K$). Here it is defined as

$$w_k^- = \begin{cases} \dfrac{1}{g_k - l_k} & \text{for the defined } \mu_k \text{ in (3),} \\ \dfrac{1}{u_k - g_k} & \text{for the defined } \mu_k \text{ in (4).} \end{cases} \tag{2.43}$$

The 'min-sum target programming' method can be used to solve the problem (2.42).

### 3.1. Solving models with targeted programming

We only ask the decision maker for information about the relative importance of the goal functions. If the decision maker cannot provide such information, the analyst can use appropriate methods based on the data from the payment table, calculate the weights of the objective functions and suggest to the decision maker to accept the solution, or can calculate a set of effective solutions obtained by varying the weights of the objective functions. Here we will present the solutions obtained by applying the proposed methodology of linearization of goal functions using the above four approaches to solve the target programming model.Primjenom modela (2.27) na gore navedene podatke linearizirane su funkcije cilja na sljedeći način:

$$\frac{\sum_{j=1}^{30} c_{kj} x_j + c_0^k}{\sum_{j=1}^{30} d_{kj} x_j + d_0^k} \leq z_k^* / \cdot (\sum_{j=1}^{30} d_{kj} x_j + d_0^k), k = 1,2,3 \text{ (goal functions that need to be}$$

maximized)

$$\sum_{j=1}^{30} c_{kj} x_j + c_0^k \le z_k^* (\sum_{j=1}^{30} d_{kj} x_j + d_0^k) \Rightarrow$$

$$\sum_{j=1}^{30} c_{kj} x_j - z_k^* \sum_{j=1}^{30} d_{kj} x_j \le z_k^* d_0^k - c_0^k \Rightarrow$$

$$\sum_{j=1}^{30} c_{kj} x_j - z_k^* \sum_{j=1}^{30} d_{kj} x_j + d_k^- = z_k^* d_0^k - c_0^k. \quad (3.1)$$

Targeted programming models look like this:

(1) Min max form

min g( $d_k^-$ ), $k$ = 1, 2, 3

$$\text{u. o. } \sum_{j=1}^{30} c_{kj} x_j - z_k^* \sum_{j=1}^{30} d_{kj} x_j + d_k^- = z_k^* d_0^k - c_0^k \quad (3.2)$$

limitations (3.1).

Model (4.8) is solved by Zimmermann's approach for solving the target programming model:

min $\lambda$

s. t. $\lambda - d_1^+ \ge 0$

$\lambda - d_2^- \ge 0$

$\lambda - d_3^- \ge 0$

$$\sum_{j=1}^{30} c_{kj} x_j - z_k^* \sum_{j=1}^{30} d_{kj} x_j + d_k^- = z_k^* d_0^k - c_0^k \quad (3.3)$$

limitations (4.4).

(2) Minimization of the sum of deviation variables:

$$\min \sum_{k=1}^{3} d_k^-$$

$$\text{u. o. } \sum_{j=1}^{30} c_{kj} x_j - z_k^* \sum_{j=1}^{30} d_{kj} x_j + d_k^- = z_k^* d_0^k - c_0^k \quad (3.4)$$

limitations (4.4).

(3) Minimization of the weight sum of deviation variables:

$$\min \sum_{k=1}^{3} w_k d_k^-$$

$$\text{u. o. } \sum_{j=1}^{30} c_{kj} x_j - z_k^* \sum_{j=1}^{30} d_{kj} x_j + d_k^- = z_k^* d_0^k - c_0^k \qquad (3.5)$$

limitations (4.4),

where $w_1 = 0.5$, $w_2 = 0.3$, $w_3 = 0.2$ are the weightiness of the goal function given by the decision maker.

Model (4.11) is solved by solving the following linear programming model

min $\lambda$

$$\text{u. o. } \sum_{j=1}^{30} c_{kj} x_j - z_k^* \sum_{j=1}^{30} d_{kj} x_j + d_k^- = z_k^* d_0^k - c_0^k$$

limitations (4.4) (3.6)

$$\lambda - w_1 d_1^- \geq 0,$$

$$\lambda - w_2 d_2^- \geq 0,$$

$$\lambda - w_3 d_3^- \geq 0,$$

where $\lambda$, $w_1$, $w_2$, $w_3$, $d_1^-, d_2^-, d_3^- \geq 0$, a $\sum_{k=1}^{3} w_k = 1$.

(4) Lexicographic form: Three goals are ranked according to their priority: goal 1 has priority 1, goal 2 has priority 2, while goal 3 has priority 3. The general lexicographic program looks like this:

$$\text{min } a = \min_{k \in P_i} g_k(d_k^-) : i = 1, 2, 3;\ k = 1, 2, 3$$

$$\text{u. o. } \sum_{j=1}^{30} c_{kj} x_j - z_k^* \sum_{j=1}^{30} d_{kj} x_j + d_k^- = z_k^* d_0^k - c_0^k,\ k = 1, 2, 3$$

limitations (4.4) , (3.7)

where $k \in P_i$ means that the $k$-th goal is at the $i$-th priority level. In our model, $z_1$ is at the first level, goal $z_2$ is at the second level, and goal $z_3$ is at the third level. Model (3.66) was solved by the simplex method in several steps.

**3.2. Solving 'fuzzy' modeling by programming**

When solving the problem of optimizing a production plan with 'fuzzy' programming, it is necessary to first make 'membership' functions for goal functions (relation (3.47), calculate the weights of goal functions (3.53) and form a target programming model (3.52) looks as follows:

$$\text{Min } \frac{1}{0.057198} D_1^- + \frac{1}{2.461424} D_2^- + \frac{1}{0.067538} D_3^-$$

u. o. limitations (4.4)

ISSN 2661-2666 (Online) International Scientific Journal Monte (ISJM) DOI: 10.33807/monte.20222618
Volume 6, (No).2 (2022): April

$$\sum_{j=1}^{30}\left(c_{1j}-0.273726d_{1j}\right)x_j+D_1^-=8173155.997$$

$$\sum_{j=1}^{30}\left(c_{2j}-5.563876d_{2j}\right)x_j+D_2^-=1344315.9 \qquad (3.8)$$

$$\sum_{j=1}^{30}\left(c_{3j}-0.256522d_{3j}\right)x_j+D_3^-=8125460.815$$

$$D_1^- -0.216528\sum_{j=1}^{30}d_{1j}x_j\le 6858631.148$$

$$D_2^- -3.102452\sum_{j=1}^{30}d_{2j}x_j\le 749598.94$$

$$D_3^- -0.188984\sum_{j=1}^{30}d_{3j}x_j\le 5986161.369$$

### 3.3. Solutions

By applying the goal satisfaction method, the following solution was obtained:

Table 4.5. Solutions by the method of meeting goals

| Solution | Values of variables | $z_1$ | $z_2$ | $z_3$ |
|---|---|---|---|---|
| (1) | $x_4$ = 249207, $x_5$ = 575000, $x_6$ = 500000, $x_7$ = 500000, $x_8$ = 172500, $x_9$ = 232156, $x_{13}$ = 500000, $x_{14}$ = 500000, $x_{15}$ = 230000, $x_{16}$ = 500000, $x_{17}$ = 500000, $x_{21}$ = 453350, $x_{22}$ = 500000, $x_{25}$ = 32636, $x_{26}$ = 152594, $x_{27}$ = 230000, $x_{28}$ = 300000, $x_{29}$ = 264500 | 0.231248 | 4.622657 | 0.228204 |

The following set of solutions was obtained by the method of target programming:

Table 4.6. Solutions by the method of target programming

| Solution | Values of variables | $z_1$ | $z_2$ | $z_3$ |
|---|---|---|---|---|

| | | | | |
|---|---|---|---|---|
| (1) Model (3.3) | $x_2$ = 157039, $x_5$ = 499792, $x_6$ = 500000, $x_7$ = 500000, $x_{12}$ = 64010, $x_{13}$ = 500000, $x_{14}$ = 45893, $x_{15}$ = 230000, $x_{16}$ = 50000, $x_{21}$ = 500000, $x_{22}$ = 500000, $x_{26}$ = 65734, $x_{27}$ = 230000, $x_{28}$ = 300000, $x_{29}$ = 264500, $x_{30}$ = 500000 | 0.24188 | 3.74423 | 0.224984 |
| (2) Model (3.4) | $x_4$ = 345000, $x_6$ = 500000, $x_7$ = 500000, $x_8$ = 172500, $x_{11}$ = 10249, $x_{13}$ = 500000, $x_{14}$ = 500000, $x_{15}$ = 230000, $x_{16}$ = 50000, $x_{17}$ = 475143, $x_{21}$ = 500000, $x_{22}$ = 500000, $x_{26}$ = 169847, $x_{27}$ = 230000, $x_{28}$ = 300000, $x_{29}$ = 264500 | 0.218005 | 4.50649 | 0.256523 |
| (3) Model (3.5) | $x_2$ = 249473, $x_3$ = 1860, $x_5$ = 575000, $x_6$ = 171910, $x_7$ = 500000, $x_{12}$ = 69444, $x_{13}$ = 446755, $x_{15}$ = 230000, $x_{16}$ = 50000, $x_{21}$ = 500000, $x_{22}$ = 500000, $x_{25}$ = 139177, $x_{27}$ = 230000, $x_{28}$ = 300000, $x_{29}$ = 264500, $x_{30}$ = 500000 | 0.252394 | 3.453361 | 0.203193 |
| (4) Model (3.6) | $x_4$ = 345000, $x_5$ = 575000, $x_6$ = 500000, $x_7$ = 212180, $x_8$ = 172500, $x_{10}$ = 113829, $x_{12}$ = 10234, $x_{13}$ = 338275, $x_{14}$ = 50000, $x_{15}$ = 230000, $x_{16}$ = 500000, $x_{21}$ = 500000, $x_{22}$ = 401730, $x_{25}$ = 500000, $x_{27}$ = 230000<br>$x_{28}$ = 300000, $x_{29}$ = 264500, $x_{30}$ = 153597 | 0.262165 | 3.742245 | 0.155269 |

The solutions obtained using the 'fuzzy' programming method are shown in the following table:

Table 4.7. Fuzzy programming solutions

| Solution | Values of variables | $z_1$ | $z_2$ | $z_3$ |
|---|---|---|---|---|

ISSN 2661-2666 (Online) International Scientific Journal Monte (ISJM) DOI: 10.33807/monte.20222618
Volume 6, (No).2 (2022): April

| | | | | |
|---|---|---|---|---|
| (1) Model (3.7) | $x_2$ = 157039, $x_5$ = 499792, $x_6$ = 500000, $x_7$ = 500000, $x_{12}$ = 64010, $x_{13}$ = 500000, $x_{14}$ = 45893, $x_{15}$ = 230000, $x_{16}$ = 50000, $x_{21}$ = 500000, $x_{22}$ = 500000, $x_{26}$ = 65734, $x_{27}$ = 230000, $x_{28}$ = 300000, $x_{29}$ = 264500, $x_{30}$ = 500000 | 0.24188 | 3.74423 | 0.224984 |

## 4. Conclusion

This paper proposes a methodology for linearization of fractional target functions to solve the MOFLP problem by the target programming method. The proposed methodology was tested on the problem of optimizing the company's financial structure.

To solve the optimal financial structure of the company, four approaches were used for the problem of targeted programming. The obtained results revealed the possibility of efficient application of the proposed methodology in solving the given problem.

There are many advantages of the proposed methodology compared to existing models:

(a) The methodology is simple for the analyst as well as for the decision maker.

(b) The decision maker may determine the weightiness of the target functions, where the solutions obtained reflect the preferences of the decision maker.

(c) This method allows the analyst to design a set of efficient solutions by varying the weightinessy of the objective functions where the decision maker can choose the preferred solution.

The presented approach is limited to solving multi-criteria fractional linear problems with unclear goals - where decision makers cannot express the desired value of goal functions.

For the next research, we propose to further expand the application of the proposed methodology that solves practical multicriteria fractional programming problems using the proposed target programming model.

## References

[1] R. Caballero, M. Hernandez, Restoration of efficiency in a goal programming problem with linear fractional criteria, European Journal of Operational Research, 172, 2006, 31–39.

[2] M. Chakraborty, S. Gupta, Fuzzy mathematical programming for multi objective linear fractional programming problem, Fuzzy Sets and Systems, 125, 2002, 335–342.

[3] A. Charnes, W.W. Cooper, Management Models of Industrial Applications of Linear Programme (Appendix B), vol.-1, Wiley, New York, 1961.

[4] B.D. Craven, Fractional Programming, Heldermann Verlag, Berlin, 1988.

[5] T. Gomez, M. Hernandez, M.A. Leon, R. Caballero, A forest planning problem solved via a linear fractional goal programming model, Forest Ecology and Management, 227, 2006, 79–88.

[6] E.L. Hannan, Linear programming with multiple fuzzy gols, Fuzzy sets and systems, 6, 1981, 235-248.

[7] J.P. Ignizio, Goal Programming and Extensions, Lexington D.C.Health. MA, 1976.

[8] C. Kao, S.T. Liu, Fractional programming approach to fuzzy weighted average, Fuzzy Sets and Systems, 120, 2001, 435–444.

[9] J.S.H. Kornbluth, R.E. Steuer, Goal programming with linear fractional criteria, European Journal of Operational Research, 8, 1981, 58-65.

[10] J.S.H. Kornbluth, R.E. Steuer, Multiple objective linear fractional programming, Management Science, 27, 1981, 1024-1039.

[11] Y.Z. Mehrjerdi, Solving fractional programming problem through fuzzy goal setting and approximation, Applied Soft Computing, 11, 2011, 1735-1742.

[12] B. Metev, D. Gueorguieva, A simple method for obtaining weakly efficient points in multiobjective linear fractional programming problems, European Journal of Operational Research, 126, 2000, 386-390.

[13] B. Mishra, S.R. Singh, Linear Fractional Programming Procedure for Multi Objective Linear Programming Problem in Agricultural System, International Journal of Computer Applications, 60, 2013, 0975-8887.

[14] R.H. Mohamed, The relationship between goal programming and fuzzy programming, Fuzzy sets and systems, 89, 1997, 215-222.

[15] H. Ohta, T. Yamaguchi, Linear fractional goal programming in consideration of fuzzy solution, European Journal of Operational Research 92, 1996, 157-165.

[16] B.B. Pal, I. Basu, A goal programming method for solving fractional programming problems via dynamic programming, Optimization, 35, 1995, 145-157.

[17] B.B. Pal, B.N. Moitra, U. Maulik, A goal programming procedure for fuzzy multiobjective linear fractional programming problem, Fuzzy Sets and Systems, 139, 2003, 395-405.

[18] M. Pašić, A. Ćatović, I. Bijelonja, A. Bahtanović, Goal Programming Nutrition Optimization Model, Annals of DAAAM for 2012 & Proceedings of the 23rd International DAAAM Symposium, , Editor Branko Katalinic, Published by DAAAM International, Vienna, Austria, 2012, pp. 0243 - 0246

[19] T. Perić, Z. Babić, Determining Optimal Production Program with Fuzzy Multiple Criteria Programming Method, Proceedings of the International multiconference of engineers and computer scientists, Hong Kong, 2009, 2006-2013.

[20] T. Perić, Z. Babić, Financial structure optimization by using a goal programming approach**,** Croatian Operational Research Review, 3, 2012, 150-162.

[21] M. Sakawa, T. Yumine, Interactive fuzzy decision making for multiobjective linear fractional programming problems, Large Scale Systems, 5, 1983, 105-114.

ISSN 2661-2666 (Online) International Scientific Journal Monte (ISJM) DOI: 10.33807/monte.20222618
Volume 6, (No).2 (2022): April
[22] Tiwary, R.N., Dharmar, S. and Rao, J.R. (1987). Fuzzy goal programming –an additive model, Fuzzy Sets and Systems, 24, 27-34.

[23] H.-J. Zimmermann, Fuzzy programming and linear programming with several objective functions, Fuzzy Sets and Systems, 1, 1978, 45-55.

[24] H.-J. Zimmermann, Fuzzy Sets, Decision Making and Expert Systems, Cluwer Academic Publishers, Boston, 1987.